

# Obtaining Robotic Object Models from the Equations of the Potential-Flow Method

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**Abstract** – Robotic systems almost always use for their work chemical current sources (CCS), the state of which directly affects the ability, duration and safety of their operation. To diagnose the technical condition of a CCS as a technical object (TO) and form laws governing them, it is necessary to know the relationship between their unobservable parameters, the observed output parameters and known external influences on them. The mentioned relations (laws of the technical object operation) can be learned from the equations of physicochemical processes in the objects under consideration, in particular, the equations of the potential-flow method [1]. In this paper a method for obtaining models of a technical object from the equations of the potential-flow method is presented.

**Index Terms** – physicochemical processes, robotic object.

## I. INTRODUCTION

CONTROL AND DIAGNOSTICS of objects of robotic complexes, including their power sources, are the most important aspects of improving various indicators of these objects, such as power density, specific capacity, efficiency of storage and converters of electrical and mechanical energy (lithium-ion batteries (LIB), hydrogen fuel cell (HFC)), including indicators of reliability and operation safety. For synthesis of control systems (CS) it is necessary to know the link between the output characteristics of TO (OCTO) (for example, output voltage, battery temperature, fuel cell, output torque, temperature inside the engine) and the external (input) influences (EI) on TO (for example, ambient temperature, current in the external circuit of the battery, hydrogen fuel cell) [2 – 5]. The control system is synthesized by the method of inverse dynamics with the above links [3 – 5]. In order to diagnose objects of robotic systems, it is necessary to know the effect of unobservable output characteristics of TO with the observed OCTO and EI on the considered TO [6]; assessment of this effect is the solution to the problem of diagnostics [6].

Thus, to solve the problems of control and diagnostics, it is necessary to obtain a multitude of links between OCTO and EI (that is, the mathematical model of TO).

Currently these mathematical models of TO are obtained either statistically (by extrapolating a large number of experimental data (ED)) or by modeling physicochemical processes (PCP) with subsequent identification of models

based on the same experimental data. The main disadvantage of the first approach is the neglect of PCP occurring in it. As a result, the mathematical model (MM) of TO turns out to be inaccurate and unsuitable for solving most practical problems. In the second approach, TO model is obtained by mathematical modeling of PCP. So the obtained TO mathematical models are more accurate; on the basis of these models, it is possible to solve control and diagnostics problems. That is why the second approach is now finding increasing use [7].

Earlier, the authors developed a potential-flow method (PFM) [12 – 14] for PCP modeling within the modern nonequilibrium thermodynamics [1, 8 – 11]. PFM is a unified approach for the mathematical description and modeling of PCPs of various physical and chemical nature [12, 13], based on the experimentally studied properties of substances and processes (PSP).

In [13], the definition of PSP from experimental data is described; due to the use of functional expansions (FE), these PSP can be determined arbitrarily within the available data sets [12, 13]. Thus, as can be seen from [12, 13], PFM can be used as the basis for a universal approach for obtaining mathematical models of technical maintenance.

This paper is devoted to the method of obtaining TO mathematical models based on potential-flow equations of the PCP dynamics in the TO test. On the basis of PFM, a unified approach is created for building TO mathematical models, on the basis of which the described control problems are solved.

## II. POTENTIAL-FLOW METHOD

PFM is a unified approach of mathematical description and modeling of processes of different physical and chemical nature. It is based on modern non-equilibrium thermodynamics [8 – 11].

In the modern non-equilibrium thermodynamics, the state of the system is characterized by state coordinates (SC) – quantities that uniquely characterize the state of the system regardless of its history, and the change of each SC is due to the PCP of the corresponding nature [8 – 12]. Internal energies (IE) of energy degrees of freedom (EDF) (IE phase, IE degrees of forward, rotational and vibrational motion of molecules [10, 12], subphase IE, having its own temperature (in case of two or more temperatures in phase) [10, 12]) are SC as well [8 – 12].

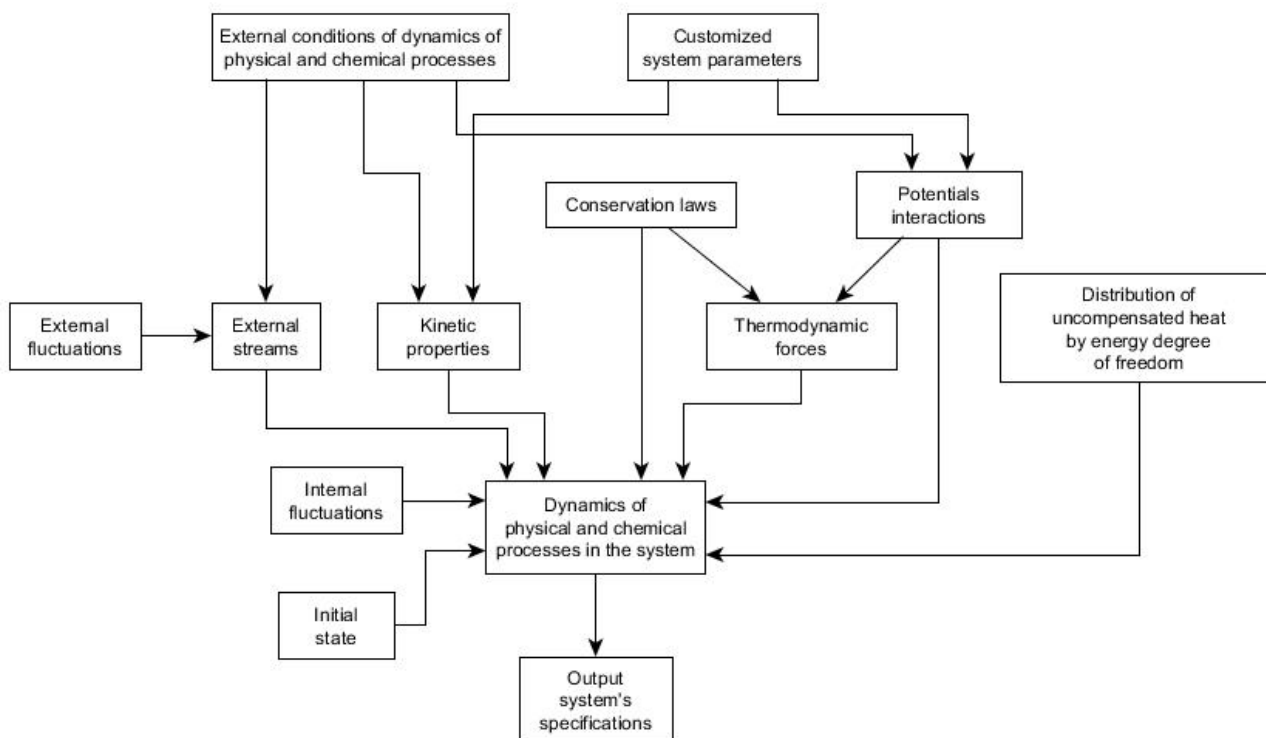


Fig. 1. Factors determining the dynamics of physicochemical processes from the standpoint of modern non-equilibrium thermodynamics.

The temporal dynamics of a PCP is characterized by the coordinates of the processes (CP) – dynamic quantities, each of which changes only as a result of the flow of the corresponding PCP. The rate of flow of PCP – the rate of change of CP. CP increments are associated with the SC increment and external flows by the balance equations resulting from the corresponding conservation laws (including the first law of thermodynamics) (Fig. 1) [8 – 12].

The cause and the necessary condition for the flow of PCPs are thermodynamic forces (TF), determined through the interaction potentials (IP) (Fig. 1) [8 – 12]. However, the TF does not uniquely determine the entire dynamics of the course of PCP; In addition to the TF, regardless of the latter, the dynamics of PCP is determined by the kinetic properties determined by the conditions of the PCP flow (Fig. 1) [12]. The “scale” of kinetic properties is the positive definite kinetic matrix (KM) [12]; flow rates of PCP are determined through the product of KM on the column of TF vector [8, 12].

The KM of the system (in which the PCP flow) is determined through the positively defined KM of its simple subsystems (sets of PCPs conjugated with each other and not conjugated to the PCP from other such sets) [12].

As a result of PCP flow, uncompensated heat is released [8 – 12], which are distributed in energy degrees of freedom in accordance with the distribution of uncompensated heat (Fig. 1) [12].

Also, the PCP dynamics in the system are determined by its internal fluctuations and fluctuations of its external flows (Fig. 1) [11].

EDF temperatures (in the general case, nonequilibrium) are special cases of interaction potentials [8 – 12].

The IP of the system and the KM of its simple subsystems are also determined by the “individual” parameters of the system (Fig. 1), which vary from instance to instance of the system and some of which are unknown.

The PCP dynamics in the system under consideration determines the dynamics of its output characteristics (OCTO dynamics) (Fig. 1) [8 – 12].

Thus, in order to obtain the OCTO dynamics at different explosives, it is necessary to know (from the ED) the following characteristics of PCP (Fig. 1) [12]: interaction potentials; kinetic matrices of simple subsystems of the system under consideration; distribution of uncompensated heat values by energy degrees of freedom; external streams; external conditions of flow of PCP; individual system parameters; the initial state of the system, and also randomly set internal to external fluctuations [12].

To determine IP and KM from the ED (as functions of the SC state and individual parameters), functional decompositions (FD) are set for these values, whose coefficients are found from the ED obtained in laboratory systems (LS) (Fig. 2) [13, 15, 16]. From these EDs are finds in parallel: the distribution of uncompensated heats by EDF; external streams (unknowns); individual system parameters (unknown); initial state of the system.

Since the individual system parameters vary from instance to instance of the system (from the LS instance to the LS instance), the necessary for determining the above values of the ED are obtained in the LS ensemble. From these EDs, in

addition to the FD coefficients for IP, KM, the distribution of uncompensated heats on the EDF, the individual parameters of each LS in ensemble under consideration, as well as the other parameters mentioned above are found (Fig. 2, 3). In further studies, only the FD factors for IP and KM are used, as well as the distribution of uncompensated heat values over the EDF. In general, it turns out from the ED only the range of possible values of these coefficients [13, 15, 16].

These values are determined from the condition [13, 15, 16]:

$$\mathbf{z}_{vsb,i,j}(t) = \mathbf{z}_{vsb,i,j}^E(t), \quad t \in T_{i,j}^E, \quad j=1, n_{u,i}, \quad i=1, N_{LS}, \quad (1)$$

where  $N_{LS}$  is number of LS in ensemble under consideration;

$n_{u,i}$ ,  $i=1, N_{LS}$  is the number of tests of  $i$ -th LS in ensemble under consideration;

$T_{i,j}^E$ ,  $j=1, n_{u,i}$ ,  $i=1, N_{LS}$  are time moments, when sensor readings from  $i$ -th LS in ensemble under consideration are taken;

$\mathbf{z}_{vsb,i,j}(t)$ ,  $\mathbf{z}_{vsb,i,j}^E(t)$ ,  $t \in T_{i,j}^E$ ,  $j=1, n_{u,i}$ ,  $i=1, N_{LS}$  are calculated and experimental values of observed OCTO.

Calculated values  $\mathbf{z}_{vsb,i,j}(t)$ ,  $t \in T_{i,j}^E$ ,  $j=1, n_{u,i}$ ,  $i=1, N_{LS}$  of observable OCTOs are determined in accordance with the PFM (Fig. 1). At the same time, the sought FD factors for KM of simple subsystems and IP, as well as the distribution of uncompensated heats for EDF are taken to be the same for all LS of the LS ensemble (Fig.3), and the individual LS parameters are “theirs” for each LS and are the same for all tests of each specific LS (Fig. 3).

Thus, the described values are sought by minimizing the objective function (OF) (Fig. 2):

$$\Delta \mathbf{z}_{vsb,i,j}(t) = \mathbf{z}_{vsb,i,j}(t) - \mathbf{z}_{vsb,i,j}^E(t), \quad (2)$$

$$t \in T_{i,j}^E, \quad j=1, n_{u,i}, \quad i=1, N_{LS},$$

$$G = \frac{1}{2N_{LS}} \sum_{i=1}^{N_{LS}} \frac{1}{n_{u,i}} \sum_{j=1}^{n_{u,i}} \sum_{t \in T_{i,j}^E} \Delta \mathbf{z}_{vsb,i,j}^T(t) A_t \Delta \mathbf{z}_{vsb,i,j}(t), \quad (3)$$

where  $A_t$ ,  $t \in T_{i,j}^E$ ,  $j=1, n_{u,i}$ ,  $i=1, N_{LS}$  is symmetric positive definite matrix.

From (2) and (3) it can be seen that the entered TF takes the smallest value equal to zero, if and only if condition (1) is fulfilled. Thus, minimizing the TF (2) and (3), the FD for the IP and KM and the distribution of uncompensated heats for the EDF (or a set of possible values of these quantities) are obtained. The basic functions of the FD for the IP and the KM are selected (from the criteria of the best approximation – the minimum of the TF (2), (3)) using symbolic regression [17 – 21].

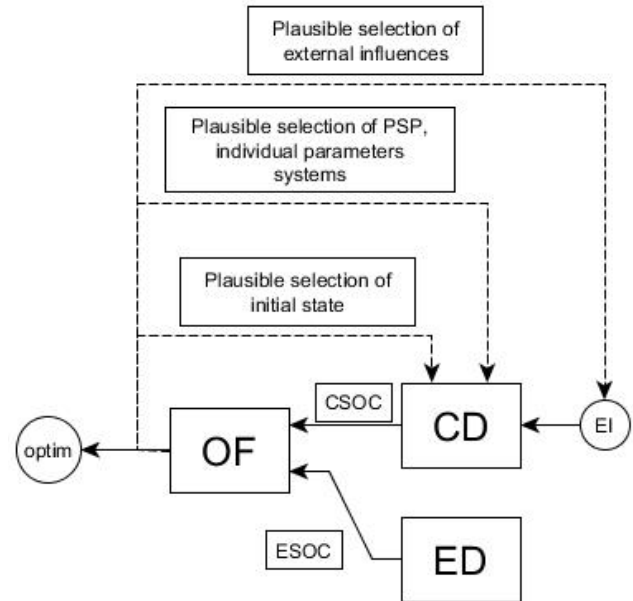


Fig. 2. The scheme of determining from ED of FD for KM of simple subsystems, IP, distributions of uncompensated heats for EDF (as well as the initial state, external flows (unknowns), individual parameters). CD is calculated dynamics; CECTO is calculated OCTO; EECTO is experimental OCTO.

### III. OBTAINING THE MATHEMATICAL MODEL OF THE ROBOTICS COMPLEX OBJECT

Let us determine from the experimental data described above and in [13] the set of possible functional expansions for the kinetic matrices of simple subsystems, interaction potentials, and also the distributions of uncompensated heats in energy degrees of freedom. And they, in turn, depend on individual parameters that vary from system to system. On these sets of possible values of individual parameters, we are looking for the MM of the investigated (or projected) TO (Fig. 4). In this case, the considered MM is sought from the condition of the coincidence of the results of the calculation of the OCTO according to this model and the calculation of the same OCTO by the PFM [16].

The desired MM of TO represents the dependence of higher-order derivatives of OCTO in time from lower derivatives of OCTO and EI and derivatives of EI:

$$\mathbf{v}_z(t) = \mathbf{f}(\mathbf{x}_z(t), \mathbf{x}_s(t), \mathbf{p}) + \mathbf{e}_z(t), \quad (4)$$

$$\mathbf{v}_y(t) = \mathbf{g}(\mathbf{x}_y(t), \mathbf{x}_z(t), \mathbf{x}_s(t), \mathbf{p}) + \mathbf{e}_y(t), \quad (5)$$

where  $\mathbf{v}_z(t)$  are upper derivatives of observed OCTO  $\mathbf{z}(t)$ ;  $\mathbf{x}_z(t)$  are the observed OCTO  $\mathbf{z}(t)$  and their lower derivatives;  $\mathbf{v}_y(t)$  are upper derivatives of unobservable OCTO  $\mathbf{y}(t)$ ;  $\mathbf{x}_y(t)$  are unobservable OCTO  $\mathbf{y}(t)$  and their lower derivatives;  $\mathbf{x}_s(t)$  are EI  $\mathbf{s}(t)$  and their derivatives;

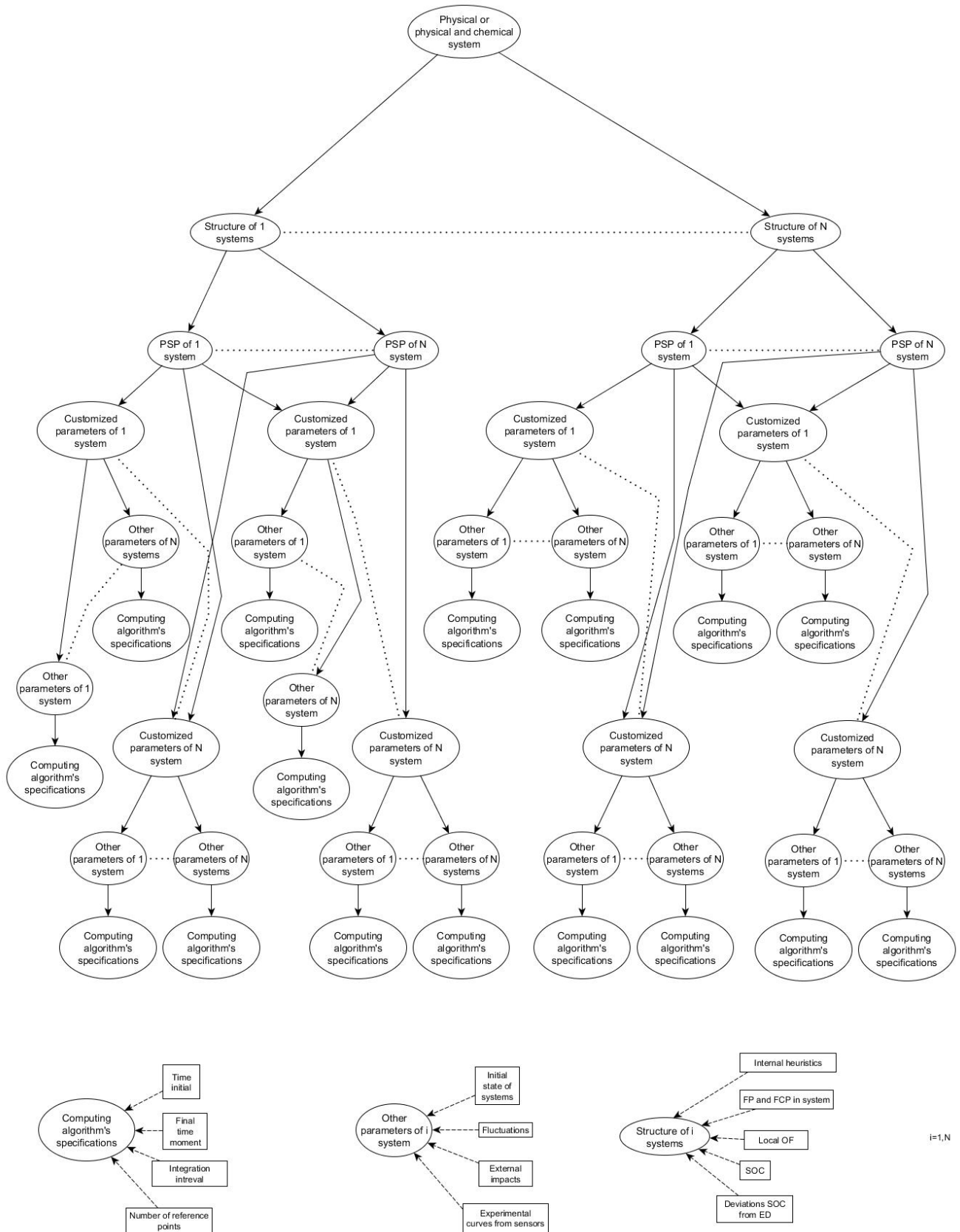


Fig. 3. Experimental studies of PSP in the LS ensemble.

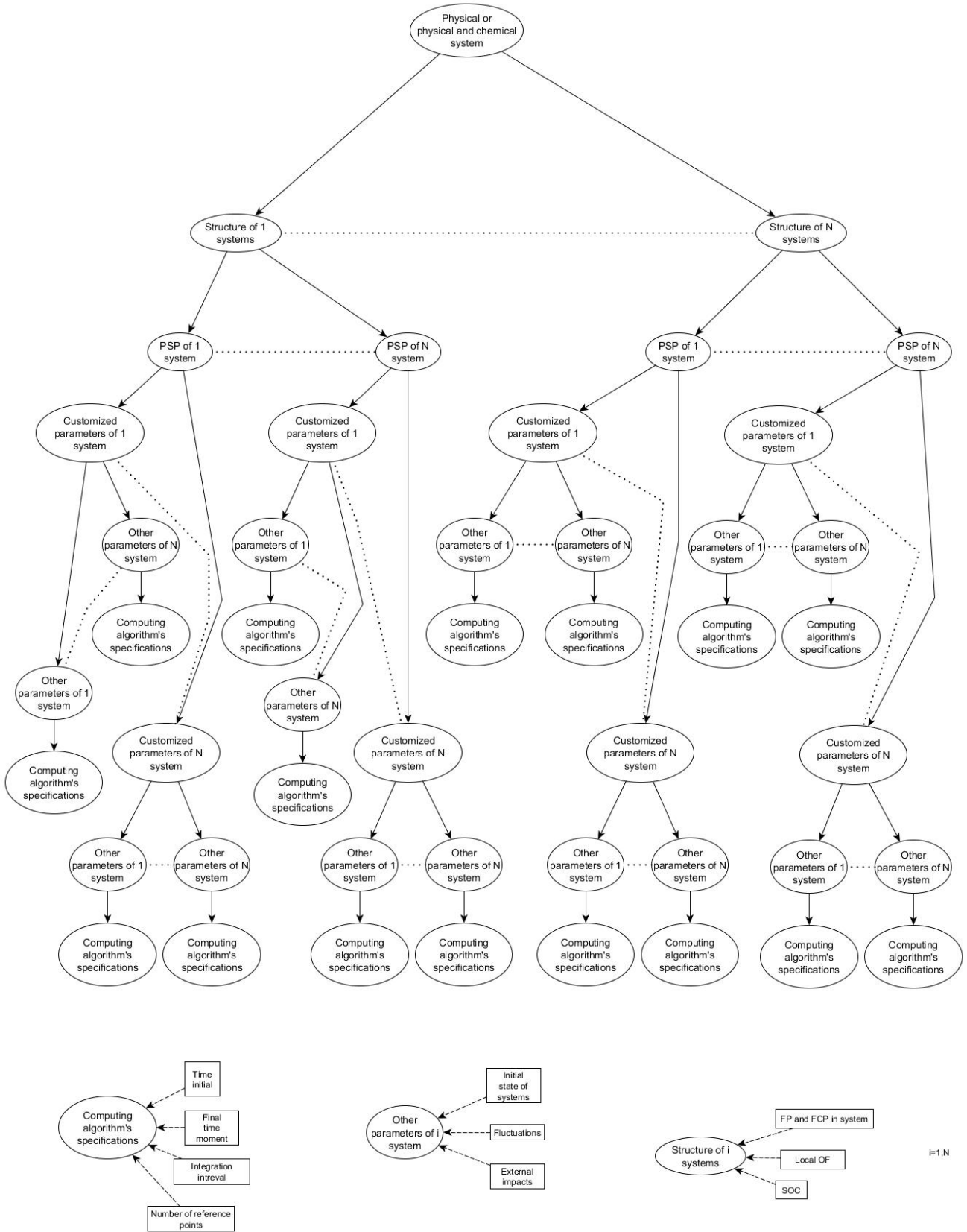


Fig. 4. Getting TO MM on the set of possible PCP dynamics.

$\mathbf{p}$  are the parameters that have a fixed value for a particular instance of TO and are different for different instances of TO;  $\mathbf{e}_z(t)$ ,  $\mathbf{e}_y(t)$  - components of noise, and:

$$\|\mathbf{e}_z(t)\| \ll \|\mathbf{v}_z(t)\|, \|\mathbf{e}_y(t)\| \ll \|\mathbf{v}_y(t)\|. \quad (6)$$

Thus, as it is easy to see from (4) – (6), the function  $\mathbf{f}(\mathbf{x}_z, \mathbf{x}_s, \mathbf{p})$  and  $\mathbf{g}(\mathbf{x}_y, \mathbf{x}_z, \mathbf{x}_s, \mathbf{p})$  are searched by symbolic regression [16 – 20] by minimizing the target function (seen from Fig. 4).

$$\Delta \mathbf{z}_{i,j}(t) = \begin{pmatrix} \mathbf{v}_{z,i,j}(t) - \mathbf{f}(\mathbf{x}_{z,i,j}(t), \mathbf{x}_{s,i,j}(t), \mathbf{p}_i) \\ \mathbf{v}_{y,i,j}(t) - \mathbf{g}(\mathbf{x}_{y,i,j}(t), \mathbf{x}_{z,i,j}(t), \mathbf{x}_{s,i,j}(t), \mathbf{p}_i) \end{pmatrix} \quad (7)$$

$$t \in T_{i,j}^*, \quad j = 1, n_{crv,i}, \quad i = 1, N_{var},$$

$$G = \frac{1}{2N_{var}} \sum_{i=1}^{N_{var}} \frac{1}{n_{crv,i}} \sum_{j=1}^{n_{crv,i}} \sum_{t \in T_{i,j}^*} \Delta \mathbf{z}_{i,j}^T(t) A_i \Delta \mathbf{z}_{i,j}(t), \quad (8)$$

where  $N_{var}$  is number of different PSP options, individual maintenance parameters;

$n_{crv,i}$ ,  $i = 1, N_{var}$  is the number of calculated dynamic OCTO curves corresponding to the  $i$ -th set of PSP and individual system parameters;

$A_i$  is a symmetric positive definite matrix.

In other words, expressions (4) and (5) under condition (6) are obtained by eliminating the SC from the dynamic equations of the PFM (Fig. 1) by the substitution method. From here, searching for functions  $\mathbf{f}(\mathbf{x}_z, \mathbf{x}_s, \mathbf{p})$  and  $\mathbf{g}(\mathbf{x}_y, \mathbf{x}_z, \mathbf{x}_s, \mathbf{p})$  by the described method of symbolic regression using (7) and (8), we take the initial symbolic approximations of these unknown functions, simplifying the dynamic equations of the PFM, and excluding the SC from the simplified dynamic equations. Then, these functions are modulated using symbolic regression [17 – 21] using (7) and (8).

It should also be noted that if the considered TO functions from a certain initial state (at the initial moment of time) in which known constraints are superimposed on the values of the SC, then we also get from the equations of the PFM and these relationships by the described methods:

$$\mathbf{x}_y(t_0) = \mathbf{q}(\mathbf{x}_z(t_0), \mathbf{x}_s(t_0), \mathbf{p}) + \mathbf{e}_q, \|\mathbf{e}_q\| \ll \|\mathbf{x}_y(t_0)\|. \quad (9)$$

If the right sides of (4) and (5) are integrated analytically, then taking the integral over the left and right sides using (9), and specifying on the set of possible OCTO dynamics (Fig. 4)  $\mathbf{e}_z(t)$  and  $\mathbf{e}_y(t)$  (their maximal absolute values), we obtain TO MM in the form of integro-differential equations.

Having received the TO MM in the described way (equations (5), (6) and (9)), we determined the coefficients  $\mathbf{p}$ , we obtain the MM of a specific TO instance, on the basis of which it is possible to synthesize control system and the diagnostic algorithm [2 – 6].

## VI. CONCLUSIONS

A method of obtaining a mathematical model of the investigated (or projected) object in the form of (4) – (6), (9) is presented. On the basis of this MM, it is possible to synthesize the control system of the methods described in [3 – 6], to form an algorithm for diagnosis using the methods described in [6]. Thus, the potential-stream method of mathematical modeling of physicochemical processes is a unified universal approach for describing PCPs of various nature and is a unified approach that allows the synthesis of control systems and diagnostic algorithms for various objects of robotic systems, including chemical sources of electricity.

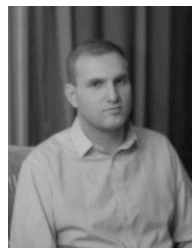
To obtain numerical solutions of control and diagnostic problems, in (4), (5), (9) it is convenient to use differential derivatives. Obtaining (4) – (6), (9) for differential derivatives is similar to that described above for derivatives. In the case of difference derivatives, performing algebraic transformations (4), (5), (9), we obtain in the general case in the right-hand side of (4), (5) a functional.

In [22], the described approach is considered in relation to lithium-ion batteries, which are currently the main source of electricity for all types of robotic systems.

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